# LOCAL HEAT TRANSFER ON THE ENTRANCE SEGMENT OF A TUBE WITH A SHARP INLET EDGE. <br> <br> 2. CORRECTION FOR THE ENTRANCE SEGMENT <br> <br> 2. CORRECTION FOR THE ENTRANCE SEGMENT UNDER TURBULENT BOUNDARY LAYER CONDITIONS 

 UNDER TURBULENT BOUNDARY LAYER CONDITIONS}

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#### Abstract

A correction relation $\varepsilon_{\mathrm{x}}=f\left(\operatorname{Re}_{\mathrm{d}} ; X / d\right)$ is proposed for the entrance segment, where turbulent flow occurs in $a$ surface boundary layer developing upon completing the transient process. An elevated local heat transfer intensity in a separated flow region beyond a sharp inlet edge is considered to be a consequence of the $M$-shaped velocity field profile that exists over a substantial length of the entrance segment.


A qualitative analysis of the axial distributions of the local heat transfer coefficients on the entrance segment of a round tube with a sharp inlet edge [1] opens the way for a physically based approach in the experimental determination of the correction $\varepsilon_{\mathrm{x}}$ in an equation adopted for short channels with $\operatorname{Re}_{\mathrm{d}}>10^{4}$ and $\mathrm{X} / \mathrm{d}<10$ [2]:

$$
\begin{equation*}
\mathrm{Nu}_{d}=0,022 \varepsilon_{x} \mathrm{Re}_{d}^{0,8} \mathrm{Pr}^{0,43} \tag{1}
\end{equation*}
$$

In accord with the conclusions of [1], generation of a turbulent boundary layer in a separated flow region beyond a sharp inlet edge takes place not at once in the cross section $\mathrm{X} / \mathrm{d}=0$ but markedly later in conformity with the completion of the transient process, when the reduced length attains the value of the secondary critical coordinate. Thus, use of formula (1) may be warranted only for $\mathrm{X} / \mathrm{d}>(\mathrm{X} / \mathrm{d})_{\mathrm{cr}}^{\mathrm{II}}$. This fact is allowed for in developing the relation for the correction $\varepsilon_{\mathrm{x}}$ discussed later.

The array of experimental data for $\alpha=\mathrm{f}\left(\mathrm{X} / \mathrm{d} ; \mathrm{Re}_{\mathrm{d}}\right.$ ) satisfying the restrictions $(\mathrm{X} / \mathrm{d})_{\mathrm{cr}}^{\mathrm{II}}<\mathrm{X} / \mathrm{d}<13$ is taken from [1], where it is shown that for a sharp inlet edge with an angle of $90^{\circ}$ and $\operatorname{Re}_{d}=(13-110) \cdot 10^{3}$ the second critical coordinate on the wall of the entrance segment is a constant and amounts to $(\mathrm{X} / \mathrm{d})_{\mathrm{cr}}^{\mathrm{II}} \approx 1.62$. The structure of the computational relation for the correction $\varepsilon_{\mathbf{x}}$ was chosen by preliminarily comparing two widely used versions

$$
\begin{gather*}
\varepsilon_{x}=c_{0}(X / d)^{n_{0}}  \tag{2}\\
\varepsilon_{x}=1+c_{1}(X / d)^{n_{1}} \tag{3}
\end{gather*}
$$

Version (2) has been adopted in [2,3]. For convenience from the point of view of the methods of primary processing of experimental findings it is usually assumed that in it either the Reynolds number does not affect the power $n_{0}$ and the coefficient $c_{0}$ or its effect fits within the framework of a small error in computations of the $\mathrm{Nu}_{\mathrm{d}}$ number.

For the configuration of the inlet edge considered, version (2) is inapplicable since the graphs of log $\left(\mathrm{Nu}_{\mathrm{d}} / \mathrm{Nu}_{\mathrm{d}, \infty}\right)=\mathrm{f}\left(\log \mathrm{Re}_{\mathrm{d}}\right)$ with $\mathrm{Re}_{\mathrm{d}}=$ const prove to be substantially nonlinear although the scatter in terms of the Reynolds number permits an averaging nonlinear approximation with an error of the order of $\pm 15 \%$. Structure (3) has been proposed by Hausen in [4] and subsequently adopted in [5-7] and in some other works devoted mainly to mean heat transfer. It provides a satisfactory linearization of the function $\log \left(\mathrm{Nu}_{\mathrm{d}} / \mathrm{Nu}_{\mathrm{d}, \infty}-1\right)=\mathrm{f}(\log \mathrm{X} / \mathrm{d})$ and is therefore chosen as the preferred version in the present article. However, processing experimental data in the coordinate system [ $\left.\left(\mathrm{Nu}_{\mathrm{d}} / \mathrm{Nu}_{\mathrm{d}, \infty}-1\right) ; \mathrm{X} / \mathrm{d}\right]$ involves difficultes associated with the smallness of the difference $\left(\mathrm{Nu}_{\mathrm{d}} / \mathrm{Nu}_{\mathrm{d}, \infty}-1\right)$ at the end of the entrance segment, where $\mathrm{X} / \mathrm{d}>8$. Here the complex $\left(\mathrm{Nu}_{\mathrm{d}} / \mathrm{Nu}_{\mathrm{d}, \infty}-1\right)$ rapidly tends to zero, which raises severely, practically by an order of magnitude, the requirements on the measuring accuracy for the local heat transfer coefficients. The latter confines the analysis of experimental results to the reduced lengths $\mathrm{X} / \mathrm{d}<8-10$.

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Fig. 1. Graph $\mathrm{Nu}_{\mathrm{d}} / \mathrm{Nu}_{\mathrm{d}, \infty}-1=\mathrm{f}(\mathrm{X} / \mathrm{d})$ for the entrance segment under turbulent boundary layer conditions.


Fig. 2. Constant $\mathrm{c}_{1}$ in Eq. (3) vs Reynolds number.
Graphs of $\left(N u_{d} / N u_{d, \infty}-1\right)=f(X / d)$ for five values of the Reynolds number are constructed in Fig. 1 and are approximated by lines with the slope $n_{1}=-1.7$. A dashed line encloses a domain with a $5 \%$ error in the $N u_{d}$ number. Judging from the extrapolated estimates in the direction of $X / d>10$, the correction $\varepsilon_{\mathrm{x}}$ increases the stabilized value of the Nusselt number less than $1 \%$ at a distance of $25-30$ tube diameters from the entrance cross section. The coefficients $\mathrm{c}_{1}$ found using Fig. 1 are presented in Fig. 2, which confirms the relatively weak effect of the Reynolds number on the value of the corrections $\varepsilon_{\mathrm{x}}$. Over the range from $\mathrm{Re}_{\mathrm{d}}=10^{4}$ to $\mathrm{Re}_{\mathrm{d}}=10^{5}$ the coefficients $\mathrm{c}_{1}$ decrease approximately $35 \%$. This adjusts the correction $\varepsilon_{\mathrm{x}}$ by $10-12 \%$. The slope of the graph in Fig. 2 is -0.14 , and hence,

$$
\begin{equation*}
c_{1}=11,3 \operatorname{Re}_{d}^{-0,14} \tag{4}
\end{equation*}
$$

or

$$
\begin{equation*}
\varepsilon_{x}=1+11,3 \operatorname{Re}_{d}^{-0,14}(X / d)^{-1,7} \tag{5}
\end{equation*}
$$

If the dependence of the correction $\varepsilon_{\mathbf{x}}$ on the Reynolds number is neglected, then with an accuracy up to $10 \%$ in the Nusselt number we have

$$
\begin{equation*}
\varepsilon_{x}=1+2,55(X / d)^{-1,7} \tag{6}
\end{equation*}
$$

Substituting expression (5) into (1), we arrive at a generalizing formula for calculating the local heat transfer coefficients in smooth tubes with an entrance segment having a sharp inlet edge with an angle of $90^{\circ}$ for $\mathrm{Re}_{d} \geq 10^{4}$, $\operatorname{Pr}=0.71$, and $X / d \geq 1.6$ :


Fig. 3. Generalization of the experimental values of $\mathrm{Nu}_{\mathrm{d}}$ for the entrance segment under turbulent boundary layer conditions by using formula (7): 1) $\mathrm{Re}_{\mathrm{d}}=$ $13 \cdot 10^{3}$; 2) $28 \cdot 10^{3}$; 3) $42 \cdot 10^{3}$; 4) $77 \cdot 10^{3}$; 5) $\mathrm{Re}_{\mathrm{d}}=110 \cdot 10^{3}$.


Fig. 4. Correction relations $\varepsilon_{\mathrm{X}}=\mathrm{f}(\mathrm{X} / \mathrm{d})$ for round tubes and $\mathrm{Re} \geq 10^{4}$ according to [2, 5-7] and Eq. (5): 1) Eq. (5) ; 2) [6]; 3) [5]; 4) [7]; 5) [2].

$$
\begin{equation*}
\mathrm{Nu}_{d}=0,019 \operatorname{Re}_{d}^{0,8}+0,21 \operatorname{Re}_{d}^{0.66}(X / d)^{-1,7} \tag{7}
\end{equation*}
$$

Results of calculations using formula (7) - solid curves and experimental values of $\mathrm{Nu}_{\mathrm{d}}$ over the reduced length range $1.6 \leq \mathrm{X} / \mathrm{d} \leq 13$ - are presented in Fig. 3. According to the data from [1], a dash denotes the location of a transition region. Calculations using formula (7) agree with experiment with a deviation of $\pm 5 \%$.

It is necessary to note one specific feature of the heat transfer process in turbulent tube flow described by formula (7). If for fixed cross sections with $\mathrm{X} / \mathrm{d}$ exceeding 1.6 the experimental data are processed by using a simple power relation $\mathrm{Nu}_{\mathrm{d}}=\mathrm{cRe} e_{\mathrm{d}}^{\mathrm{p}}$, then each of them has its own exponent within the range $0.66<\mathrm{p}<0.8$. For example, p $=0.72$ near the critical coordinate $(\mathrm{X} / \mathrm{d})_{\mathrm{cr}}^{\mathrm{II}}$, and the exponent $\mathrm{p}=0.8$ is realized only for $\mathrm{X} / \mathrm{d} \geq 20$. A similar regular trend is seen in [3]. Hence, stabilization of the local heat transfer intensity on the entrance segment is accompanied by a monotonical approach of the exponent of the Reynolds number to its upper limit $\mathrm{p}=0.8$, characterizing developed flow.

TABLE 1. Correction Equations for Entrance Segments of Round Tubes Having a Sharp Inlet Edge with an Angle of $90^{\circ}$

| Reference | Equation |
| :---: | :---: |
| $[5]$ | $\varepsilon_{\mathrm{x}}=1+0.9\left(\frac{X}{\mathrm{~d}}\right)^{-1}$ |
| $[6]$ | $\varepsilon_{\mathrm{x}}=1+\frac{\mathrm{d}}{\mathrm{X}}\left(\frac{6 \cdot 10^{3}}{\operatorname{Re}_{\mathrm{d}}}+0.06 \mathrm{Re}_{\mathrm{d}}^{0.25}\right)$ |
| $[7]$ | $\varepsilon_{\mathrm{x}}=1+\left(1.55-0.2 \operatorname{logRe} \mathrm{e}_{\mathrm{d}}\right)\left(\frac{\mathrm{d}}{\mathrm{X}}\right) \frac{29.6}{\operatorname{Re}_{\mathrm{d}}^{0.54}}+0.107 \frac{\mathrm{X}}{\mathrm{d}}$ |

Comparative results using Eq. (5) and equations cited in [5-7], where the local heat transfer in tubes having an identical configuration of the inlet edge (see Table 1) was investigated, are presented in Fig. 4. Curve 5, which is recommended in [8] as a universal relation for the correction $\varepsilon_{\mathrm{x}}$, on entrance segments of tubes for $\mathrm{Re}_{\mathrm{d}} \geq 10^{4}$, is plotted there using the relation $\varepsilon_{\mathrm{x}}=1.38(\mathrm{X} / \mathrm{d})^{-0.12}$ taken from [2]. Numerical calculations using [5-7] and Eq. (7) are close. Here, attention should be concentrated not on quantitative but on qualitative differences in the physical interpretation of the correction formulas. The separation of curve 5 downward relative to curves $1-4$ may be attributed only to the nonidentical flow conditions on the entrance segment in experiments in [5-7] and in [2]. In [2], the local heat transfer coefficients were measured beyond Witoshinskii's nozzle, which provided a practically uniform initial velocity profile. At the same time, beyond sharp edges, as shown in [9] and [10], an M-type velocity profile with a sharply pronounced maximum develops near the external boundary of the wall boundary layer. At a maximum, the local velocity exceeds the mean flowrate one by $1-2$ times up to $X / d=1.5-2.5$. This is the main reason for so appreciable a local heat transfer enhancement on the portion of the entrance segment with a sharp inlet edge that lies beyond the cross section where secondary flow rises.

## NOTATION

X , longitudinal coordinate, mm ; d, diameter of the flow-type part of the stand, $\mathrm{mm} ; \mathrm{X} / \mathrm{d}$, relative longitudinal coordinate; $\varepsilon_{\mathrm{x}}$, correction factor; $\mathrm{Nu}_{\mathrm{d}}$, Nusselt number; $\mathrm{Re}_{\mathrm{d}}$, Reynolds number; Pr, Prandtl number. Subscripts: cr, critical; $\infty$, developed flow; $x$, local value; d, similarity numbers based on the diameter of the flow-type part of the stand.

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